IMO Algebra Questions

Level: Intermediate   Ref No: M02   **Puzz Points: 10**

Mars, his wife Venus and grandson Pluto have a combined age of 192. The ages of Mars and Pluto together total 30 years more than Venus' age. The ages of Venus and Pluto together total 4 years more than Mars's age. Find their three ages.

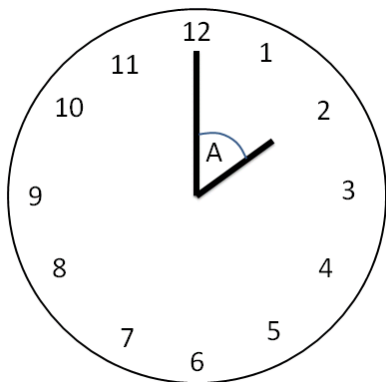**Level: Intermediate   Ref No: M09**   **Puzz Points: 15**

A $s \times s$ square, where s is an odd integer, is divided into unit squares (1×1). All the unit squares along the edges and the two diagonals of the $s \times s$ square are discarded. Find a fully simplified expression, in terms of s, for the number of unit squares remaining.

Level: Intermediate   Ref No: M11   **Puzz Points: 18**

    (a) What is the angle A between the hands of the clock at 2 o'clock?
    (b) What is the next time after this that the angle between the hands is equal to A?

Level: Intermediate   Ref No: M14   **Puzz Points: 20**

An *arithmetic* sequence is one in which the difference between successive terms remains constant (for example, 4, 7, 10, 13, …). Suppose that a right-angled triangle has the property that the lengths of its sides form an arithmetic sequence. Prove that the sides of the triangle are in the ratio 3:4:5.

Level: Intermediate   Ref No: M16   **Puzz Points: 20**

Solve the simultaneous equations:

$$x + y = 3$$
$$x^3 + y^3 = 9$$

Level: Intermediate   Ref No: M23   **Puzz Points: 13**

Mij the magician has a large bag of red balls and a large bag of green balls. Mij wanders round the audience selecting volunteers, asking each volunteer to remove two balls, one from each bag, until $\frac{2}{5}$ of the red balls and $\frac{3}{7}$ of the green balls have been removed. The balls remaining in the bags are then emptied into a bucket. What fraction of all the balls does the bucket contain?

Level: Intermediate   Ref No: M24   **Puzz Points: 13**

A mathematician has a full one-litre bottle of concentrated orange squash, a large container and a tap. He first pours half of the bottle of orange squash into the container. Then he fills the bottle from the tap, shakes well, and pours half of the resulting mixture into the container. He then repeats this step over and over again: filling the bottle from the tap each time, shaking the mixture well, and then pouring half of the contents into the container.

Suppose that on the final occasion he fills the bottle from the tap and empties it completely into the container. How many times has he filled the bottle from the tap if the final mixture consists of 10% orange squash concentrate?

Level: Intermediate   Ref No: M27   **Puzz Points: 15**

James, Alison and Vivek go into a shop to buy some sweets. James spends £1 on four Fudge Bars, a Sparkle and a Chomper. Alison spends 70p on three Chompers, two Fudge Bars and a Sparkle. Vivek spends 50p on two Sparkles and a Fudge Bar.

What is the cost of a Sparkle?

Level: Intermediate   Ref No: M29   **Puzz Points: 18**

The Principal of Abertawe Academy plans to employ more teachers. If she employs 10 new teachers, then the number of pupils per teacher will be reduced by 5. However, if she employs 20 new teachers, then the number of pupils per teacher will be reduced by 8.

How many pupils are there at Abertawe Academy?

Level: Intermediate   Ref No: M32   **Puzz Points: 20**

Find all integer values that satisfy the following equations:

$$x^2 + y^2 = x - 2xy + y$$
$$x^2 - y^2 = x + 2xy - y$$

Level: Intermediate   Ref No: M38   **Puzz Points: 10**

Before the last of a series of tests, Sam calculated that a mark of 17 would enable her to average 80 over the series, but that a mark of 92 would raise her average mark over the series to 85.

How many tests were there in the series?

Level: Intermediate   Ref No: M54   **Puzz Points: 23**

Find all real values of *x* and *y* that satisfy the equations:

$$x^4 - y^4 = 5$$
$$x + y = 1$$

Level: Intermediate   Ref No: M57   **Puzz Points: 10**

At dinner on a camping expedition, each tin of soup was shared between 2 campers, each tin of meatballs was shared between 3 campers and each tin of chocolate pudding was shared between 4 campers. Each camper had all three courses and all tins were emptied.

The camp leader opened 156 tins in total. How many campers were on the expedition?

Level: Intermediate   Ref No: M61   **Puzz Points: 15**

If Julie gave £12 to her brother Garron, then he would have half the amount that she would have. If instead Garron gave £12 to his sister Julie, then she would have three times the amount that he would have.

How much money do they each have?

Level: Intermediate   Ref No: M83   **Puzz Points: 18**

Find all solutions to the simultaneous equations:

$$x^2 - y^2 = -5$$
$$2x^2 + xy - y^2 = 5$$

Level: Intermediate   Ref No: M90   **Puzz Points: 10**

An aquarium contains 280 tropical fish of various kinds. If 60 more clownfish were added to the aquarium, the proportion of clownfish would be doubled.

How many clownfish are in the aquarium?

Level: Intermediate   Ref No: M92   **Puzz Points: 10**

Two difference rectangles are placed together, edge-to-edge, to form a large rectangle. The length of the perimeter of the large rectangle is $\frac{2}{3}$ of the total perimeter of the original two rectangles.

Prove that the final rectangle is in fact a square.

Level: Intermediate   Ref No: M98   **Puzz Points: 15**

Four positive integers a, b, c, and d are such that:

- The sum of a and b is half the sum of c and d.
- The sum of a and c is twice the sum of b and d.
- The sum of a and d is one and a half times the sum of b and c.

What is the smallest possible values of $a + b + c + d$?

Level: Intermediate   Ref No: M100   **Puzz Points: 18**

Two different cuboids are placed together, face-to-face, to form a large cuboid. The surface area of the large cuboid is $\frac{3}{4}$ of the total surface area of the original two cuboids.

Prove that the lengths of the edges of the large cuboid may be labelled x, y and z, where

$$\frac{2}{z} = \frac{1}{x} + \frac{1}{y}$$

Level: Intermediate   Ref No: M101   **Puzz Points: 20**

Five numbers are arranged in increasing order. As they get larger the difference between adjacent numbers doubles.

The average of the five numbers is 11 more than the middle number. The sum of the second and fourth numbers is equal to the largest number.

What is the largest number?

Level: Intermediate   Ref No: M103   **Puzz Points: 20**

Solve the simultaneous equations

$$\frac{5xy}{x+y} = 6$$

$$\frac{5xz}{x+z} = 3$$

$$\frac{3yz}{y+z} = 2$$

Level: Intermediate   Ref No: M118   **Puzz Points: 20**

How many different ways are there to express $\frac{2}{15}$ in the form $\frac{1}{a} + \frac{1}{b}$, where a and b are positive integers with $a \le b$?

Level: Intermediate   Ref No: M120   **Puzz Points: 20**

Solve the equations:

$$x + xy + x^2 = 9$$
$$y + xy + y^2 = -3$$

Level: Intermediate   Ref No: M135   **Puzz Points: 18**

Four friends, Anna, Bob, Claire and Duncan, all have different heights and the sum of their heights if 6m 72cm. Anna is 8cm taller than Claire, and Bob is 4cm shorter than Duncan. The sum of the heights of the tallest and shortest of the friends is 2cm more than the sum of the heights of the other two. Find the height of each person.

Solve the simultaneous equations:

$$p + pr + pr^2 = 28$$
$$p^2r + p^2r^2 + p^2r^3 = 224$$